

On Diversification: a gentle introduction of the QDX diversification approach

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Diversification matters...

- » Volatility re-appeared in financial markets. This time, a virus triggered its return but who knows what will trigger the next spike. If you are a long-term investor, keen to harvest the risk-premia of the asset class you are invested in, diversification is not just something you need to be aware of, it is something you need to master in your investment process

- » *“It’s often said that diversification is the only ‘free lunch’ available to investors; meaning that a properly diversified portfolio reduces total risk without necessarily sacrificing expected return. However, achieving true diversification is easier said than done, especially when we don’t fully know what we mean when we’re talking about diversification. While the qualitative purpose of diversification is well-known, a satisfactory quantitative definition of portfolio diversification is not.”* Flint et al. (2015)
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2767436

- » *“Diversification is at the core of portfolio selection in Modern Portfolio Theory (MPT), yet there exists no formal definition for this concept and, as a result, no unique measure to quantify the degree to which a given portfolio is diversified. Developing an ideal measure of portfolio diversification is therefore an active research area in investment management”* Carmichael et al. (2015)
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2610814

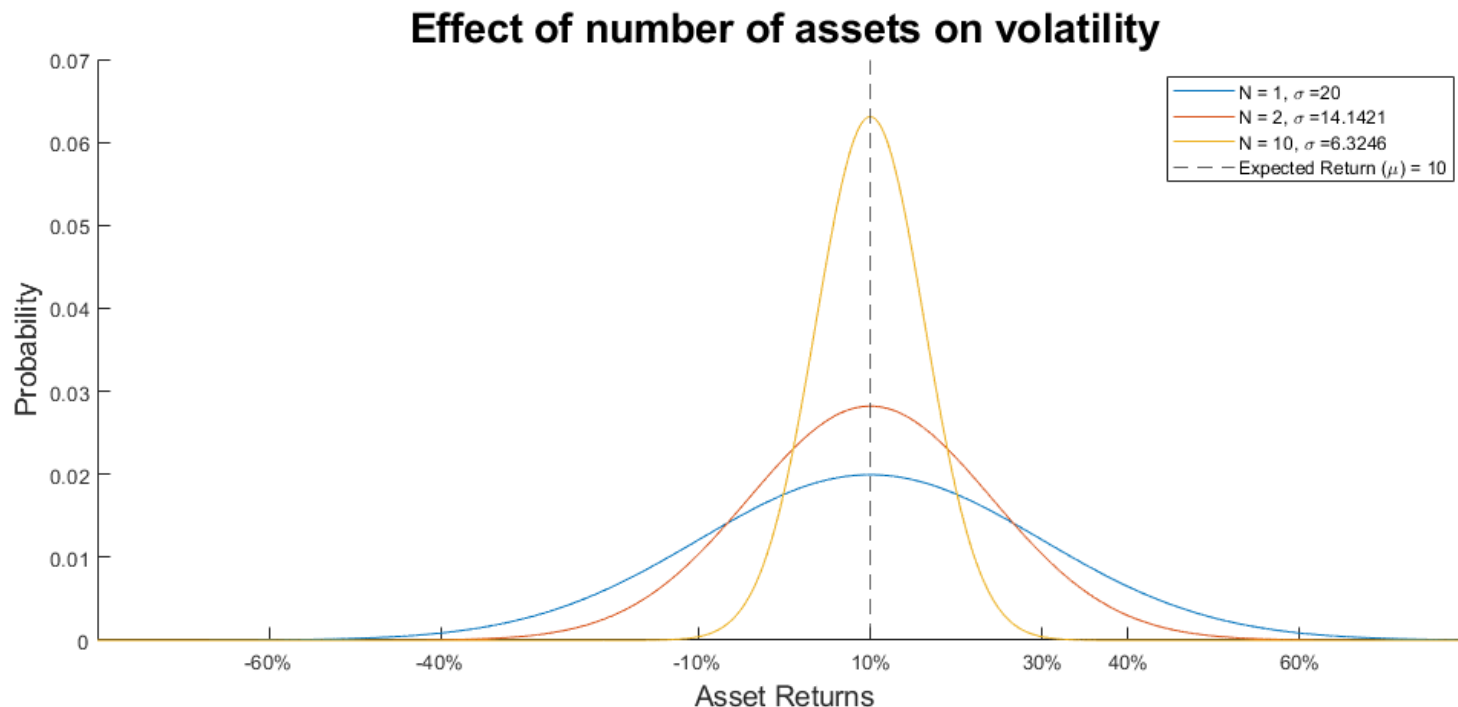
“Diversification and globalization are the keys to the future” **Fujio Mitarai, CEO of Canon Inc.**

“As a consequence, geneticists described evolution simply as a change in gene frequencies in populations, totally ignoring the fact that evolution consists of the two simultaneous but quite separate phenomena of adaptation and diversification” **Ernst Mayr, Evolutionary Biologist**

“The beauty of diversification is it's about as close as you can get to a free lunch in investing.” **Barry Ritholtz, CIO of Ritholtz Wealth Management**

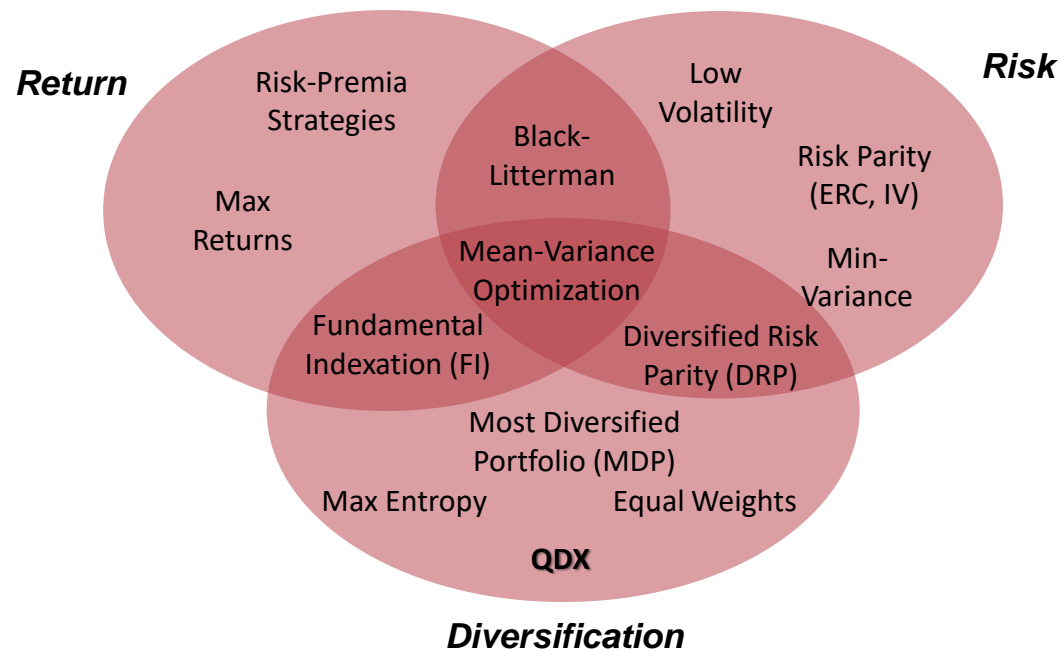
Visualising the benefits of diversification

Assuming we have an investable universe with an infinite number of **uncorrelated securities**, all sharing the same expected return (**10% per annum**) and the same volatility (**20% per annum**). The graph below exhibits what happens when the number of portfolio constituents increases. The expected return remains the same (10% per annum) but the dispersion of the return decreases to zero in the case of an infinite number of assets in the portfolio



Taxonomy of smart beta strategies

- » Diversification remains a highly researched topic in finance and we believe this contributed to move investors towards Smart Beta & Factor Investing solutions



‘Smart Beta’ is a catch all. Systematic strategies do not necessarily share the same investment goal, and even if they do it can be achieved in different ways

Introducing the QDX

- » At QIA we developed a proprietary measure of diversification
- » It measures how much volatility "savings" comes from stock interactions
- » This measure of diversification has been branded as QDX (Qatar Diversification Index), and it aims to:
 - » quantify how much risk can be diversified by analysing how portfolio constituents interact with each other
 - » its measure varies between 0 (poorly diversified) and 1 (perfectly diversified)
 - » it will tend towards 1, in a situation where portfolio constituents exhibit negatively correlation
- » Our efforts in building and implementing the QDX aim to:
 - » achieve a highly diversified portfolio, to help us during any possible future market turmoil
 - » without losing the attention to the mid term investment goal which is to deliver positive alpha versus conventional market cap benchmarks

To avoid “maths”



Diversify Diversification

May 29, 2019

Abstract

Diversification is a core concept in Asset Management. Yet, this term can mean different things to different people and there is no general consensus on how it is measured nor is there a broadly accepted metric for its reporting. In this paper, we propose a new index to quantify the diversification of a portfolio. Specifically, using the Euler's formula, we outline a new risk decomposition of the portfolio volatility that it is the basis to calculate our new diversification index (QDX). The appealing feature of our measure is its computational simplicity, given that it only requires the covariance matrix of asset returns and the portfolio allocation. The main insight beneath the QDX index is that diversification is related not to the covariance among asset returns, but to their partial covariances once we control for the portfolio return. Motivating examples supporting the use of the new measure are discussed in detail. In particular, we motivate the construction of portfolios diversifying the QDX index across assets, exploiting the entropy principle. We also provide a mathematical rationale for our index, so that it can be easily extended to homogeneous risk measures not strictly related to the portfolio volatility.

KeyWords: Diversification Measure, Portfolio Allocation, Risk Contribution, Euler decomposition

JEL code: G10, G11, D81

The QDX aims to become a new way to quantify diversification within a portfolio of risky assets

- » Three inputs are required to compute the QDX: portfolio weights and the covariance matrix (volatilities and correlations)
- » The main feature of the QDX is to decompose portfolio volatility into:
 - » an undiversified volatility, which comes with the stock itself
 - » a diversification component, which is a function of the interaction with the other portfolio holdings
- » Diversification is fueled by low correlation but it is achieved only when the variance of the residual returns, with respect to the portfolio return, is minimized

Searching for superior diversification is not an art, but a science which focuses on two features of investments: volatility and correlation

Forthcoming in the Journal of Risk:

Fusai et al. (2020), “Equally Diversified or Equally Weighted?”

https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3628585

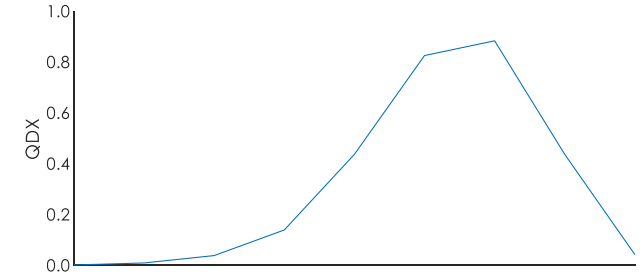
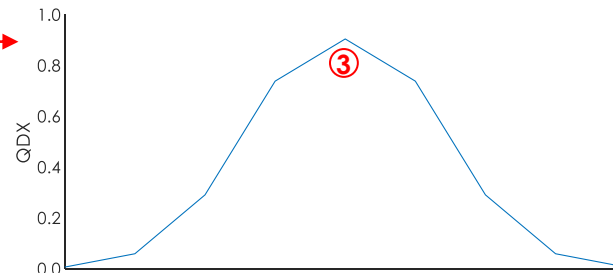
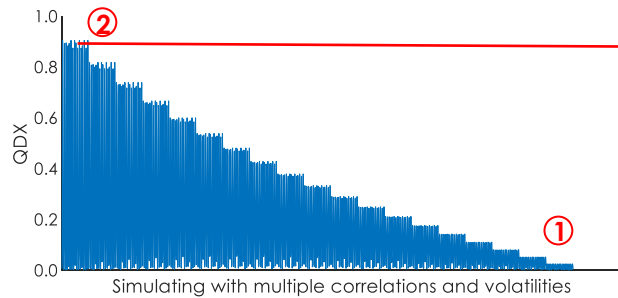
Understanding the QDX

Portfolio invested in just two assets:

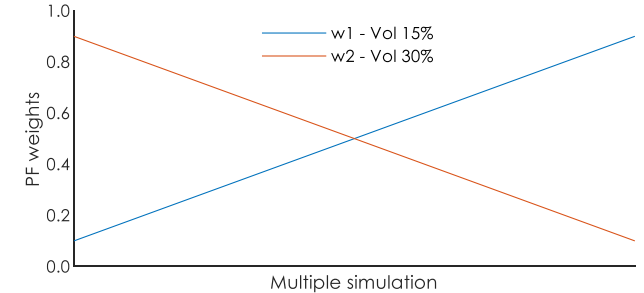
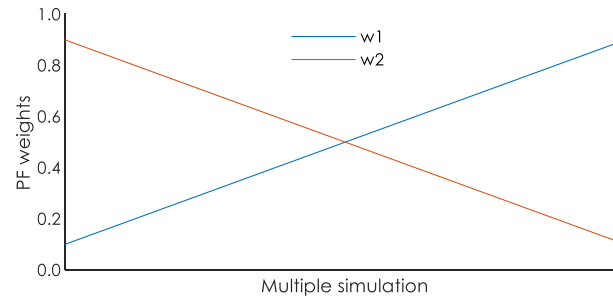
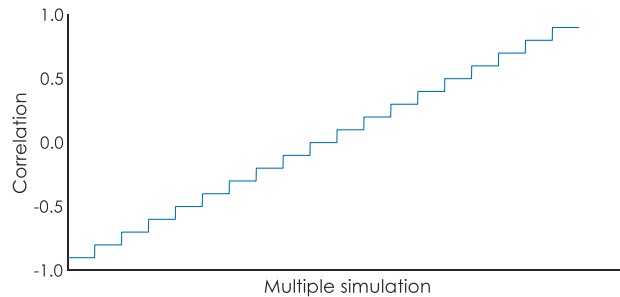
There is a direct relationship between the level of correlation and the QDX. In a perfectly positively correlated environment it is impossible to achieve diversification ①...

.... However, this does not imply that with negatively correlated securities you can achieve diversification ②. Assuming negative correlation (-.9) and assuming both stocks have the same volatility (15%) the max QDX will be achieved by the equally weighted portfolio ③.

When volatilities are not identical, a portfolio overweighted in the least volatile stock will have the highest QDX.



»Hence correlation alone is not enough



Inverse volatility, Risk Parity and QDX parity strong ties!!!!

Return of asset “i” driven by the portfolio performance (**diversified return**) and by the Alpha (stochastic **undiversified return** and **systematic**)

$$r_i = \alpha_i(w) + \beta_i(w)r_p(w) + \epsilon_i(w),$$

Weight each return by the weight (w_i) the security has in the portfolio

$$[A] w_i r_i = w_i \alpha_i + w_i \beta_i r_p + w_i \epsilon_i,$$

$$\sum_i w_i \alpha_i = \sum_i w_i \epsilon_i = 0 \text{ and } \sum_i w_i \beta_i = 1$$

From equation [A] we can recover the variance of the weighted return of asset “i” (see previous quoted article for the proof):

$$[B] \text{var}(w_i r_i) = \gamma_i^2 + \sigma_{i,p}^2$$

Where γ_i^2 is the square of the risk contribution of asset “i” and $\sigma_{i,p}^2$ is the so called residual variance. Now we have the strong link with IV, RP and QDX

$$\text{IV implies that } \text{var}(w_i r_i) = \text{var}(w_j r_j)$$

$$\text{RP implies that } \gamma_i = \gamma_j$$

$$\text{QDX implies that } \sigma_{i,p}^2 = \sigma_{j,p}^2 \quad (QDX_i = \frac{w_i^2 \sigma_i^2 - \gamma_i^2}{\sigma})$$

Comparing diversification methods

Simulation: random selection of 32 stocks from the MSCI ACWI

Analytics/Model	MV	MDR	EW	IV	RP	QDX
(a) Volatility	17.20	18.71	23.02	22.23	21.49	22.68
(b) QDX	3.95	3.91	1.05	0.99	1.14	0.93
Gross Volatility (a)+(b)	21.15	22.62	24.07	23.22	22.63	23.62
Stdev of sum Partial Variances	8.25	8.55	4.91	4.69	4.95	4.60
QDX Entropy	6.31	8.17	29.33	31.26	26.63	32.00
Risk Entropy	8.52	12.75	30.92	31.54	32.00	30.74
Weight Entropy	8.52	12.53	32.00	31.51	30.83	31.35

Even though we can see “savings” in volatility, using different entropy measures, we note that MV and MDR look more concentrated than a portfolio constructed using other techniques

QDX shows higher concentration than other indicators

We use the MSCI World Developed index to compare weight, risk and QDX concentration. Using QDX, concentration is highlighted as follows:

- Currency → 93% in the US
- Sectors → 80% in TMT + Amazon

Currency	Weight	Risk	QDX
AUD	2%	2%	0%
CAD	3%	4%	1%
CHF	3%	2%	1%
DKK	1%	0%	0%
EUR	10%	10%	2%
GBP	4%	5%	1%
HKD	1%	1%	0%
ILS	0%	0%	0%
JPY	7%	5%	1%
NOK	0%	0%	0%
NZD	0%	0%	0%
SEK	1%	1%	0%
SGD	0%	0%	0%
USD	66%	69%	93%
total	100%	100%	100%

GICS Sector	Weight	Risk	QDX
Communication Services	9%	8%	8%
Consumer Discretionary	11%	12%	26%
Consumer Staples	8%	5%	3%
Energy	3%	5%	2%
Financials	12%	16%	3%
Health Care	14%	11%	6%
Industrials	10%	11%	2%
Information Technology	22%	21%	48%
Materials	4%	4%	1%
Real Estate	3%	3%	1%
Utilities	3%	3%	1%
total	100%	100%	100%

← Amazon 22%

MSCI World Equity performance driven by APPLE, MSFT and Amazon



The ex-post performance analysis for the period August 2019 through August 2020 reinforce the findings of the QDX: APPLE, MSFT and AMAZON were the major performance drivers showing that the index is more concentrated than it appears from weights and risk decomposition. These assets together with Tesla and Facebook account for less than 10% of the index but for 50% of the performance over the period.

Sector	Performance contr.
› › Communication Services	11%
› › Consumer Discretionary	20%
› › Consumer Staples	3%
› › Energy	-8%
› › Financials	-6%
› › Health Care	19%
› › Industrials	5%
› › Information Technology	54%
› › Materials	5%
› › Real Estate	-2%
› › Utilities	0%
Grand Total	100%

Total performance was around 16.81% (Source Bloomberg)